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**AN INVENTORY DESIGN  
USING  
STAND EXAMINATIONS  
FOR PLANNING AND  
PROGRAMING  
TIMBER MANAGEMENT**

**Albert R. Stage and Jack R. Alley**



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**Albert R. Stage and Jack R. Alley**

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## ABSTRACT

Considerations guiding the design of a forest inventory for providing "in place" data for planning and programing timber management are discussed. A design is described for an inventory intended to be transitory between previous inventories that provided estimates only of forest totals and later inventories that could use a complete forest record of "in place" data. This inventory procedure uses field examination of stands in sample subcompartments augmented by aerial photo interpretation of conditions in compartments and subcompartments not examined on the ground. The data are compiled to provide timber information that can be stratified with respect to land use zones, size of stand, accessibility, land stability, productivity, and by restrictions imposed by competing uses. Estimation formulae for totals, means, and their variances are provided for the design being discussed.

# INTRODUCTION

The scope of forest inventory data--its detail to support analysis of complex silvicultural alternatives, its relevance to nontimber uses of the forest, and its utility in the everyday task of implementing management plans--is changing rapidly as increasing demands for forest benefits open new options to the forest manager. The inventory design described in this report is one of several attempts to improve the data framework for effective timber management.

Specifically, we have attempted to improve the utility of the inventory for evaluating alternative land uses and management practices in these respects:

1. Describe conditions on tracts of land that are the real units of forest land management. These tracts must be small enough that analysis of the data can direct the manager's attention to specific portions of his forest for more intensive program planning. On the other hand, the tracts must have adequate detail over an area large enough to encompass those adjacent land, vegetation, and use conditions that modify or influence our management of the tract. This is our concept of the term "in place" inventory data.

2. Provide an inventory base that can support frequent revisions of plans to show changes wrought by land management activities, and to accommodate the changing multiple use status of the various tracts that results from increased knowledge of and insight into values and needs for all forest resources.

Our objective in this report is to record a procedure we have devised that incorporates these improvements and to present some of our reasons for selecting this approach from the numerous design alternatives that were considered.



# PAST INVENTORY DESIGN

The precursor of this design was a two-stage approach very similar to that described in "Developing the data framework for effective timber management" (USDA Forest Serv. 1962). The first stage of the former inventory consisted of an extensive sample of about 200 ground-examined locations. At each ground location, a cluster of 10 sampling points was established to select trees for measurement. The ground samples were stratified by estimates of volume per acre as interpreted on aerial photos at a scale of 1:15,840. Stratum weights were estimated by photo interpretation of several thousand points systematically located on the aerial photos.

The second stage of the data framework consisted of examination of selected stands that had been defined on the aerial photos by discontinuities in texture (reflecting stocking and crown-size differences) or apparent tree height. The selection process at this stage was highly subjective because the purpose of the examination was to locate stands that would be likely candidates for particular programs--thinning, cleaning, overwood removal, etc. Ground examination of the stands selected used a series of sampling points along a transect within the stand.

In past practice, the timber management plan has been based entirely on the first stage inventory. Volumes of timber to be harvested and acreages of stands to be assigned various silvicultural treatments were estimated from the first stage sample. The second stage of stand examination became a procedure for verifying on the ground whether a particular treatment was indeed needed, and for presenting the manager with a limited number of alternative areas on which to assign priorities for a particular programed action based on expected costs and benefits.



There were several deficiencies in the past inventory design that limited its usefulness for intensive forest management. First, the ground sample locations described only a 1-acre portion or less of the stand in which the sampling location occurred. For several aspects of management, this area is inadequate. For example, silvicultural prescriptions based on such a limited portion of the stand often differed from the prescription that would have resulted had the entire stand been sampled, and had its relation to adjacent stands and other land-use determinants been described. For another example, the cover conditions of watersheds were inadequately described under the former design. To evaluate the impact of vegetation changes on runoff requires data that describe the forest cover in terms of the extent of stands and their spatial relations. As a final example, the ecological habitat was often inadequately classified from 10-point location data. In areas of disturbed vegetation, or where spatial variability characterizes the distribution of species, an area substantially larger than a single acre is required if the habitat classification is to be reliable. Hence, statistics compiled from widely scattered locations provided a misleading picture of the forest management opportunities.

The second major weakness was that the information from the second stage--the stand examination phase--was ignored in developing the estimates for the working circle. One reason for ignoring the stand examination data was that the detail of the measurements and their quality were not up to the standards of the first stage. Another reason was that the procedure by which stands were selected for examination provided no way to express the particular stand as a sample of the working circle. Though it would have been theoretically possible to assign examined stands to one subpopulation and the remainder of the forest area to another, there was no way to clearly define the subpopulations after stand boundaries had been obliterated by management practice. The opportunity to concentrate the second stage of stand examination on classes of stands of particular management interest (e.g., young dense stands in need of thinning) had been proposed as a desirable feature of the two-stage approach. In practice, it appears that the stand examination procedure was not a means to seek out new opportunities, but rather a means to verify the manager's existing impressions of the forest conditions.

# THE REVISED DESIGN FOR SAMPLING COMPARTMENTS

In an effort to correct these deficiencies, we developed a different inventory system for trial use beginning in 1970 on two National Forests, the Coeur d'Alene in north Idaho, and the Lewis and Clark in west central Montana. Development of the system was a joint endeavor of the Northern Region and the Intermountain Forest and Range Experiment Station. While this system is new to the Northern Region, other National Forest Regions or other forest owners or managers have probably used some of the same principles or techniques.

## Objectives

To support the timber management planning purpose of "providing orderly and sustained guidance for developing the timber growing capacity of working circles," we set the following specific objectives for the revised inventory design:

1. To provide "in place" data on timber management opportunities that can be stratified by land use zones, size of stand, accessibility, land stability, productivity, and by restrictions imposed by competing uses.
2. To provide "in place" data for immediate use in program development and execution.
3. To complete a timber land record system that augments the existing stand examination so as to account for all the forested acres.
4. To provide empirical information for the ecological classification of the forest into habitat types (Daubenmire and Daubenmire 1968).
5. To provide information on a stand basis that permits stands to be projected into the future, at varying intensities of management, for both inventory and management planning purposes.
6. To meet the requirements of nationwide Forest Survey and to provide inventory data compatible with other timber inventories.



# Sampling Design

The revised inventory procedure is similar to its predecessor in that it uses photo interpretation of a large number of photo points to stratify the forest acreage into classes.<sup>1</sup> However, whereas former inventories stressed volume strata, the strata used here emphasize conditions that indicate certain opportunities for timber management. When the acreages within the strata are compiled compartment by compartment, the distribution of timber management opportunities over the entire forest, or for variously defined smaller planning units, can be compiled.

Our objectives for this inventory system emphasize the importance of examining a recognizable stand in relation to the surrounding conditions of the forest lands and their uses. This emphasis is realized by defining the ground sampling unit to be a subcompartment--the smallest permanently defined record unit suitable for long-range land management planning, including a comprehensive road system. For the Coeur d'Alene National Forest, a subcompartment encompasses an area of about 1,200 acres defined by topographic, drainage, and political boundaries. The list of 604 subcompartments and their acreages that make up the Coeur d'Alene National Forest provides a logical sampling framework for not only timber management planning inventories, but for many other kinds of inventories as well. Watershed conditions, recreational use and developmental opportunities, range condition, and many other data needs that would benefit from coordination with knowledge of timber conditions could use this same overall sampling framework.

Within the selected sample subcompartments, stands are mapped from the aerial photographs and examined on the ground using a systematic grid of plots. Each of these plots is established and data recorded exactly as they have been under the former design. The location which was sampled by a 10-point cluster under the former design, is directly analogous to the stand under the revised procedure; hence, this revised procedure should retain all the capabilities of the former design.

## *SELECTION OF SAMPLE SUBCOMPARTMENTS*

The subcompartments to be included in the field sample were selected with probability proportional to their total National Forest land area. In other words, subcompartments were arrayed in order of size and systematically selected in such a manner that every acre on the forest had an equal chance to be included in the sample. The specific details of this procedure for drawing a randomized sample have been described by Stage (1971).

Because we had no past experience with sampling variation between subcompartments, we were "in the dark" as to the intensity of field sampling that would be needed to provide sufficient accuracy for the many kinds of analysis to which the data would be subjected. So four possible sample sizes were selected. The first sample included 21 subcompartments, distributed throughout the forest. The succeeding larger sample sizes included the next smaller size as a subset. Thus, the sample could include 21, 28, 34, or 40 subcompartments. By selecting four possible sample sizes, we could terminate the inventory at four possible levels depending on the rate of production or the variability encountered. Even with the smallest sample, the volume estimates should meet the standards that have been specified in the past for forest-wide totals. Whether the accuracy is appropriate for the decisions to which these data relate is a question that has yet to be satisfactorily analyzed.

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<sup>1</sup>If facilities for easily storing, modifying, and compiling mapped information were available, this stratification could be based on stand mapping on the photos with several ensuing advantages.



In the early stages of our planning, we considered optimizing the selection of subcompartments for estimating the acreages of particular categories of stands. In effect, this procedure would have resulted in as many independent, optimized inventories as there were categories of stands. Such a procedure would be desirable if the stands *not* of the category for which the subcompartment had been selected could be ignored.

We rejected this approach because the stand boundaries between categories are not always clearly defined nor would they be drawn in advance of the selection of subcompartments for examination. Thus, any bias generated by special attention to a category in drawing the boundaries would result in bias in the final estimates of the population totals.

Consequently, we decided to sample *all* stands in each selected subcompartment. By so doing, any shift in stand boundaries would *not* affect the estimates of the population totals, although the acreages in particular categories would reflect any net bias in the drawing of their boundaries.

To illustrate the distinction between these two alternatives, consider a hypothetical subcompartment consisting of two stands classified into different strata. If the expansion factor for a unit of land area sampled in one stand is the same as that for a sample in the other stand, then shifting the boundary between stands can only increase the average volume per acre (or of any other character) of one stratum at the expense of the other stratum. Consequently, the net effect on the estimate of the population total is zero. However, if the expansion factors of the two stand categories differed as they would if the sampling intensity were varied so as to optimize the independent inventories, then there would be no guarantee that the shift in boundaries would be compensating in its effect on the population total.

Moreover, including all stands in the management unit enhances our capability to consider the limitations or opportunities imposed by adjacent stand conditions.

We would like to emphasize that our reservations about sampling stands using unequal probabilities would disappear if we had the capability to map all stands in all compartments in advance of the sample selection. Forthcoming modifications of this design will, we hope, utilize complete stand mapping.

#### PHOTO SAMPLING PROCEDURE

The aerial photo phase of the inventory is accomplished using resource photography of 1:15,840 scale (4 inches per mile). Photo interpretation assigned stands (or non-stocked areas) within the selected subcompartments and grid points on the remaining area into one of 11 strata. These strata are defined on page 7. Stand boundaries are defined by discontinuities in stand height, stand texture, or crown closure. The stand is subordinate to the stratum, so that stand boundaries do not cross stratum boundaries.

Within each subcompartment selected for field sampling, all stands are delineated on the photos to a 10-acre minimum size. Their acreages are determined either by planimeter, or by counting the ground plots that fall within the stand boundary.

On the remaining portions of the forest, an overlay grid is superimposed on alternate photos to establish photosample points. The spacing of the grid points is such that each point represents about 100 acres. The interpreter then classifies the stand in which the grid point is located (not just the condition at the point) into one of the 11 strata. Along with the known acreage of the compartment, the proportion of points in a stratum establishes the stratum area.

*Definition of photo interpretation strata used in 1970 inventory on  
the Coeur d'Alene and the Lewis & Clark National Forests<sup>1</sup>*

*Stand Height Greater Than 40 Feet*

<u>Code</u>	<u>Crown Cover Description</u>	<u>Crown Cover Texture</u>
21	Medium and well stocked	Coarse <sup>2</sup>
22	Medium and well stocked	Fine
23	Poorly stocked	Coarse
24	Poorly stocked	Fine
25	Two storied--overstory with manageable understory Overstory generally poor but no more than medium stocked Understory poorly (or more) stocked: at least 100 trees per acre would be left after removing overstory	

*Stand Height Less Than 40 Feet*

26	Medium and well stocked	Fine texture
27	Poorly stocked	Fine texture
28	Apparently nonstocked	

*Other*

40	Noncommercial forest
60	Nonforest
92	Water (noncensus)

<sup>1</sup>This scheme for stand classification on aerial photos was devised by C. W. Brown, Northern Region, USDA Forest Service.

<sup>2</sup>Texture is a visual interpretation of the impression given by variability of crown size.

In addition to the timbered characteristics of each photo point, the following attributes are recorded:

- a. Multiple Use Zone and Key Value
- b. Special Study Area Classification
- c. Soil Stability Class
- d. Geographic Identity (subcompartment)
- e. Ecological Habitat Type

Estimates of the acreages of the 11 strata occurring within a particular land management unit--whether it be a Ranger District, a compartment, or any geographic area of special interest--will be computed from the proportions of the photo points falling in the respective strata.

#### *GROUND SAMPLING PROCEDURE*

A referenced base line is laid out through each selected subcompartment. The base line provides starting points for line-plot sampling of the entire subcompartment. Starting points along the base line are established for permanency and referenced for remeasurement. Transects are run perpendicularly to the base line at intervals of 10 chains horizontal distance. Plots along the transects are spaced at 5-chain intervals. This frequency provides a sample point for each 5 acres of area. All plots are referenced to the stand in which they occur as delineated on the photos. Figure 1 is a schematic diagram of the grid layout.

The inventory field procedure at each point is similar to individual points in the 1-acre 10-point cluster system. Some features of this procedure are:

- a. A fixed plot 1/300 acre in size for recording trees less than 5 inches d.b.h.
- b. A variable plot for recording trees 5.0 inches and larger.
- c. Azimuth recorded to each tree over 5.0 inches d.b.h.
- d. Each tree over 3.0 inches d.b.h. tagged.
- e. On nonstocked areas, satellite plots (1/300 acre) are taken 1/2 chain ahead, 1/2 chain back, and 1/2 chain perpendicularly to each side of the principal point of the transect. Variable plots are only taken at the principal plot location.

In addition to the typical tree data normally recorded on the plot that provide volume and condition information, inventory crews record the ecological situation as represented by vegetative indicators. We are using the Daubenmires' (1968) habitat type classification for north Idaho and eastern Washington. The inventory classification of habitat type will permit us to develop criteria for mapping the entire forest by habitat types.

#### *NET VOLUME SUBSAMPLING PROCEDURE*

Since 1964, net volume has been estimated in the Northern Region using a destructive subsampling procedure. This procedure of dissecting felled trees provides adjustment factors for three sources of error in estimates of net volume: (1) volume tables; (2) cull and defect allowances; and (3) inaccurate height estimation.

The subsample has been comprised of randomly selected locations. Under the new approach, the same procedure will be followed by drawing the subsample from the 215 locations established in the preceding inventory of 1961-62. Although the remeasurement of these sample plots will provide useful data on growth and mortality in addition to volume-table and cull and defect allowances, it will not provide control of measurement error in the current inventory. Accordingly, the destructive sampling will extend to a subsample of the points being established in the subcompartments of the current inventory.

The field procedures for the destructive sample and subsequent net tree volume calculations are described by Stage and others (1968).



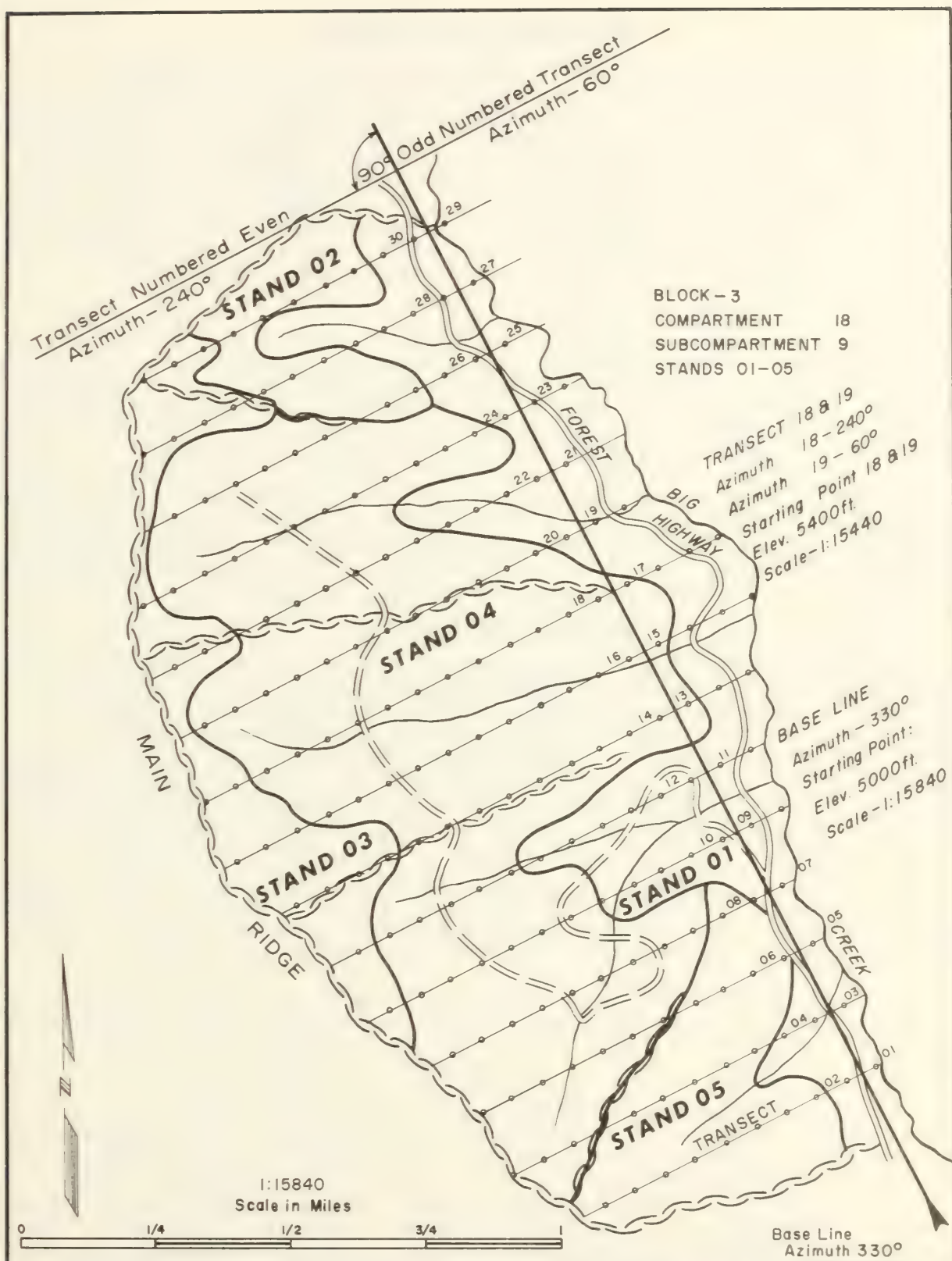


Figure 1.--Diagrammatic layout of a subcompartment with base line, transects, and sampling points.

# Remeasurement

Permanent field locations established by this inventory lend themselves to remeasurement. However, several considerations should govern the reinventory design. First, reinventories based to a large degree on repeated examination of the same sample locations will perpetuate deviations about the true population total. This persistence is particularly serious when the programed level of harvest is one of the population totals being estimated. The feedback effect of remeasured plots in inventories for harvest scheduling has not been examined thoroughly. However, there exists a definite possibility that the serial correlation of sampling errors from inventory to reinventory could result in persistent overcutting, undercutting, or unstable, destructive oscillations that would vitiate our reliance on compensating corrections as a result of successive revision of the management plan.

The second consideration is that the availability of "in place" information will play a major role in timber management program development during the execution of the plan. In other words, the timber programs will tend to be concentrated in the hundreds of stands already inventoried on the ground. Therefore, at the time of reinventory, those subcompartments already inventoried on the ground will have to be treated as one stratum, those not already inventoried on the ground as another stratum.

In summary, the design discussed in this paper should be considered a transitional design for establishing a complete timberland record based on stand examinations. Subsequent inventories for the same forest should be designed to use the information contained in these records so as to provide statistically sound estimates of the current forest situation with greatly increased efficiency.

## Compilation

Compilation procedures are quite similar to those used in processing typical random plot inventory data, the major change being development of "stand" data rather than "plot" or "location" data. The only real change in compilation procedures is to replace the constant 10 for points per cluster with a variable number of points per stand. Variance estimates are, of course, quite different. Variance computations are described on page 15. An abbreviated flow chart is included in figure 2 to illustrate data flow and the correlation of the four major types of information--the field inventory, the photo inventory of stands, photo mortality samples, and the felled-tree (net volume) data.

The first stage of the output that is of use to the forest manager is the stand analysis. These stand records are the basic material from which the management program is built. In particular, growth prognoses under alternative stand treatments can be used to develop a rational basis for establishing the management program.

It should be emphasized at this point that stand or "in place" data developed by field inventory in this overall system lose considerable utility unless the forest manager has an information system that provides ready storage and retrieval of individual stand data.

The various stand attributes are averaged within their respective strata. These stratum means are inferred to the nonsampled acres within the strata by compartments. Thus, the photo classification phase of the double-sampling procedure will extend the stand data to those compartments not sampled on the ground. Compartments average about 5,700 acres in size, so there will be about 57 photo samples in each. Compartment statistics determined solely from photo stratification will fill in the gaps in the inventory of the entire forest. As additional field inventory work (stand examination) is completed in ensuing years, it will replace information based on only strata means. The summary of these compartment records provides the final stage of the output. This output consists of a series of tables that provides the base for broad scale timber management planning.

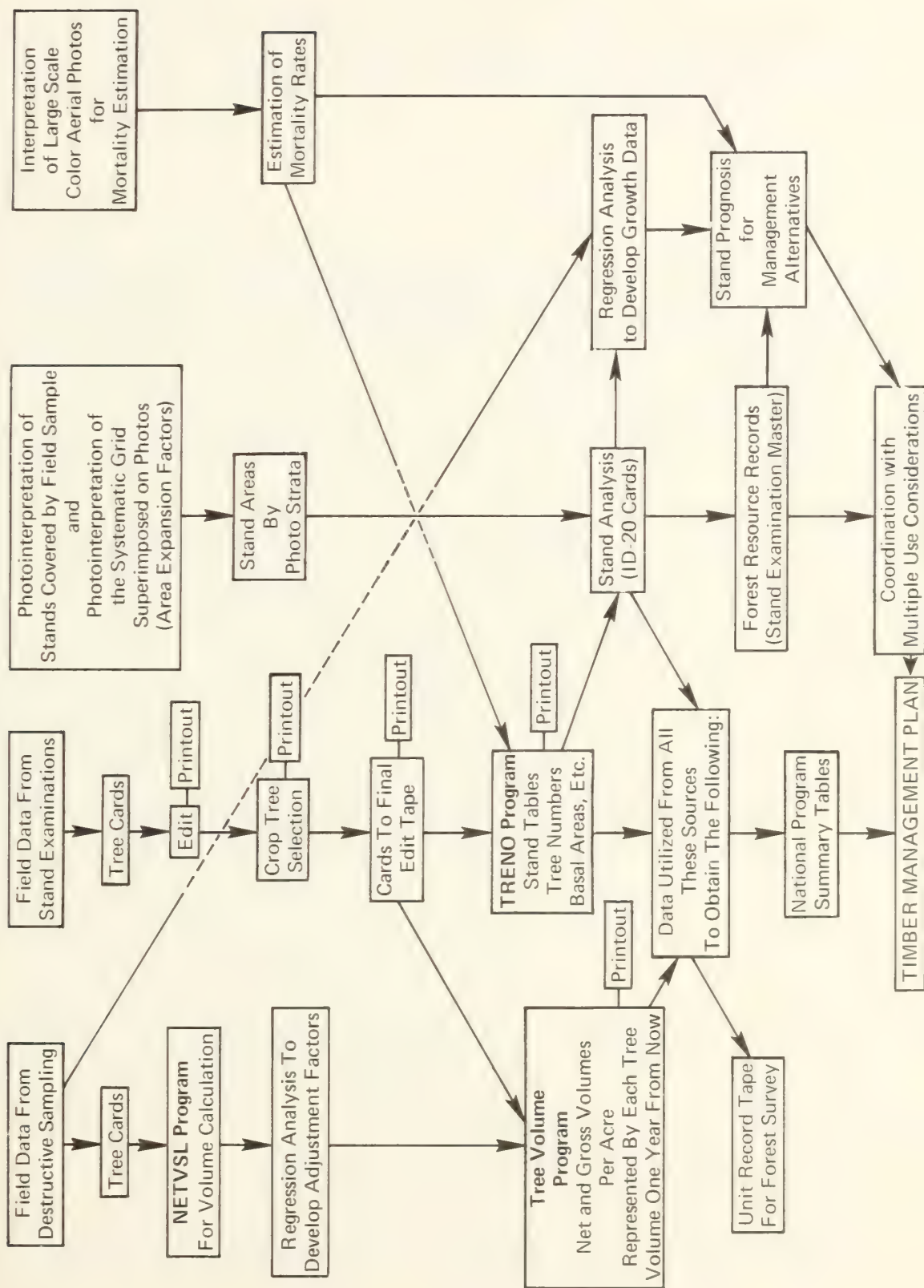


Figure 2.--Flow chart of Northern Region timber management planning data system.



## SAMPLE ESTIMATING FORMULAE

We wish to emphasize the development of estimates for compartments and sub-compartments, with forest totals developed from sums over the smaller subpopulations. Accordingly, the following expressions will differ slightly from those that might be derived solely for forest totals.

### *Summaries of Tree Characteristics Within Stands*

Let {character} be any characteristic of the sample unit that is being summarized. At the tree level, it could be the volume of the tree or any portion of it. If number of trees per acre is the item of interest, then {character} would be unity at the tree level. To obtain breakdowns by species, or tree class, {character} would be defined only for the species or class being summarized and zero otherwise.

$n_k$  = number of points in stand "k"

$c_{kj}$  = {character} of  $j$ th tree in stand "k"

$g_{kj}$  = basal area of the  $j$ th tree in stand "k"

$C_{ik}$  = total of {character} per acre of the  $k$ th stand in subcompartment "i"

baf = basal area factor for point sampling (sq.ft./tree)

fpa = fixed plot area (acres)

$$C_{ik} = \frac{1}{n_k} \sum_j \frac{c_{kj}}{f_{kj}} \quad (1)$$

where

$$f_{kj} = \begin{cases} \frac{g_{kj}}{\text{baf}} & \text{if dbh} \geq 5 \text{ in.} \\ \text{baf} & \\ \text{fpa} & \text{if dbh} < 5 \text{ in.} \end{cases}$$

### *Summaries of Stand Characteristics in Subcompartments Selected for Ground Examination*

$A_{ik}$  = area of the  $k$ th stand in subcompartment "i"

$\delta_{hik} = \begin{cases} 1 & \text{if } k\text{th stand in subcompartment "i" is in the photo stratum "h"} \\ 0 & \text{if not in that stratum} \end{cases}$

$TVA_{hi}$  = total of {character} per acre in the subcompartment "i" for only those stands in photo stratum "h"

$$M_{hi} = \frac{\sum_k A_{ik} \delta_{hik}}{\sum_k A_{ik}} = \text{proportion of area of subcompartment "i" that is in the "h" stratum}$$

$$TVA_{hi} = \frac{\sum_k n_k^C \delta_{hik}}{\sum_k n_k \delta_{hik}} \quad (2)$$

Note that the denominator is proportional to the total acreage of stratum "h" in subcompartment "i." If acres by management class are the feature being compiled,  $\delta_{hik}$  would be unity for stands in that class and zero otherwise.

#### Stratum Means

An estimate for stratum means will be developed first for the general case in which subcompartments are selected with arbitrary probabilities. The simplification that results from defining the selection probability to be proportional to acres in the subcompartment will then be demonstrated.

Let  $P_i$  be  $N$  times the probability that subcompartment "i" would be drawn for any one of the  $N$  sample units. Then  $\sum_i P_i = N$  where "i" ranges over all subcompartments in the forest. The expression for the stratum mean is then:

$$\overline{TVA}_h = \frac{\sum_i TVA_{hi} M_{hi} \sum_k A_{ik} / P_i}{\sum_i M_{hi} \sum_k A_{ik} / P_i} \quad (3)$$

Expression (3) is a fraction in which the numerator is an estimate of the total {character} in stratum "h." The denominator is the corresponding estimate of the area in the stratum. Because both numerator and denominator are estimated using the same sampling units, their covariance should help to reduce the variance of the ratio. If  $P_i$  is defined to be proportional to the total forest acreage in the  $i$ th subcompartment ( $\sum_k A_{ik}$ ), then (3) reduces to:

$$\overline{TVA}_h = \frac{\sum_i TVA_{hi} \cdot M_{hi}}{\sum_i M_{hi}} \quad (4)$$

If the proportions  $M_{hi}$  are established from absolute measures of acreage of each stand, then (3) can also be expressed as:

$$\overline{TVA}_h = \frac{\sum_i TVA_{hi} \sum_k A_{ik} \delta_{hik} / P_i}{\sum_i \sum_k A_{ik} \delta_{hik} / P_i} \quad (3a)$$

Likewise, an expression exactly equivalent to (4) can be obtained by omitting the value of  $P_i$  from (3a).

## Totals for Compartments

$CT_p$  = compartment total of {character} in all strata

$F_{hp}$  = count of photo points in stratum "h" in compartment "p" (exclusive of ground-sampled subcompartments)

$X_p$  = acres of forest land in compartment "p" (exclusive of ground-sampled acreage)

$$CT_p = \sum_h X_p \frac{F_{hp}}{\sum_l F_{lp}} \cdot \overline{TVA}_h + \sum_i TVA_{hi} \sum_k A_{ik} \delta_{hik} \quad (5)$$

In expression (5),  $i$  ranges over only the sampled subcompartments within compartment "p."

## Forest Totals

Combining all the preceding estimation formulae into an estimate of the forest grand total ( $FGT$ ),

$$FGT = \sum_p \sum_h X_p \frac{F_{hp}}{\sum_l F_{lp}} \frac{\overbrace{\sum_i M_{hi} \frac{\sum_k \delta_{hik} \sum_j c_{kj} / f_{kj}}{\sum_k n_k \delta_{hik}}}^{\overline{TVA}_h}}{\sum_i M_{hi}} + \sum_i TVA_{hi} \sum_k A_{ik} \delta_{hik} \quad (6)$$

where

$j$  indexes individual trees

$k$  indexes the stands

$i$  indexes the subcompartments

$p$  indexes the compartments

$h, l$  index the strata

For purposes of error calculation, there are two random variables in the sampling procedure. These are designated  $W$  and  $\overline{TVA}_h$  in expression (6).

$W$  is the estimated stratum weight. The expression for the variance of  $FGT$  must include a component arising from  $W$  because it is an estimate that is subject to sampling variation. The usual derivations of variance-estimating formulae for two-stage sampling plans assume that the stratum weights follow a multinomial distribution. In the design we have described, the systematic nature of the photo-point grid will result in estimates having somewhat lower variance. Williams (1956) has demonstrated how the variance of the mean of systematic samples such as these can be evaluated. However, the planned spacing between photo points in the compartments is large enough that the correlation between strata identified at points separated by 4/10 mile should be quite low. Accordingly, the multinomial distribution should be a reasonable basis for evaluating the variance of the first-stage sample of stratum weights.



The second random variable ( $\overline{TVA}_h$ ) is the estimate of stand characters for the stratum: volume per acre, excess trees per acre, silvicultural prescription, or whatever is the character associated with a sampling point in the subcompartments selected for ground examination. For purposes of assessing the variance of the stand estimates, the mean square deviation from the stand average is pertinent. However, for assessing the variance of the forest summaries over the compartments, it should be recognized that  $\overline{TVA}_h$  is the ratio of two random variables.

### Compartment Variances

The variance computations can best be illustrated by considering the compartment estimates on a per-acre basis:

$$\frac{\sum_i \frac{F_{hp}}{F_{lp}}}{\overline{TVA}_h}$$

To simplify notation and to facilitate cross-reference to standard sampling texts such as Cochran (1963), let

$n_{hi}$  = number of sample points in stratum "h" in subcompartment "i"

$$Y_{hi} = TVA_{hi} Z_{hi}$$

$$Z_{hi} = M_{hi} \sum_k A_{ik}/P_i$$

$$\begin{aligned} \bar{Y}_h &= \sum_i TVA_{hi} M_{hi} \sum_k A_{ik}/P_i \\ &= \sum_i TVA_{hi} Z_{hi} \end{aligned}$$

$$\bar{Z}_h = \sum_i M_{hi} \sum_k A_{ik}/P_i = \sum_i Z_{hi}$$

That is,  $\bar{Y}_h$  and  $\bar{Z}_h$  are numerator and denominator of  $\overline{TVA}_h$  respectively. Denote the variance of their ratio by

$$S^2_h = \text{Var } (\bar{Y}_h/\bar{Z}_h).$$

The variance of such a ratio can be approximated by the relation

$$S^2_h = \left( \frac{\bar{Y}_h}{\bar{Z}_h} \right)^2 \left( \frac{\text{Var } (\bar{Y}_h)}{\bar{Y}_h^2} + \frac{\text{Var } (\bar{Z}_h)}{\bar{Z}_h^2} - \frac{2 \text{Cov } (\bar{Z}_h, \bar{Y}_h)}{\bar{Y}_h \bar{Z}_h} \right) \quad (7)$$

With clustered samples such as are used in this design, the variance depends on the mean-square deviation of the subcompartment means ( $\bar{Y}_h, \bar{Z}_h$ ) about the corresponding mean for the forest. This mean-square deviation includes two components of variation: (1) the variance between subcompartments, and (2) the contribution from the sampling within the subcompartment that yields an estimate of the mean that differs from the true and unknown mean for that subcompartment.

To illustrate the computation of the appropriate mean-square deviation, the formula for  $\text{Cov} (\bar{Z}_h, \bar{Y}_h)$  will be used. The corresponding formulae for  $\text{Var} (\bar{Y}_h)$  or  $\text{Var} (\bar{Z}_h)$  are obvious by recalling that  $\text{Var} (\bar{Y}_h) = \text{Cov} (\bar{Y}_h, \bar{Y}_h)$ .

$$\text{Cov} (\bar{Y}_h, \bar{Z}_h) = \frac{1}{N-1} \left[ N \sum_{i=1}^N (Y_{hi} Z_{hi} / P_i^2) - \bar{Y}_h \bar{Z}_h \right] \quad (8)$$

A sample estimate of the variance for double sampling designs is computed from an approximation given by Cochran (1963, p. 333). Let

$$T_p = \sum_h F_{hp} = \text{total photo-interpreted points in compartment "p"}$$

$$CA_p = \sum_h F_{hp} \overline{TVA}_h / T_p = \text{average of \{character\} per acre in compartment "p"}$$

Then

$$\widehat{\text{var}} (CT_p) = \frac{X^2}{T_p^2} \sum_h \left[ (F_{hp}^2 + F_{hp}) S_h^2 + F_{hp} (\overline{TVA}_h - CA_p)^2 \right] \quad (9)$$

The finite population correction designated as  $g'$  by Cochran has been omitted in expression (9) because the sampling, although usually systematic, should be considered as occurring with replacement. As discussed previously, obtaining  $F_{hp}$  from a systematic sample will cause the variance estimate above to be too large if the interpretation points are so close together that their serial correlation is substantial.

The variance estimate (9) assumes that the stratum areas are estimated by the same sampling intensity in all compartments. For those compartments containing sub-compartments for which stratum areas can be determined more exactly, it would seem reasonable to determine the total areas from the sums of the known areas (dividing by acres per photo point) plus the usual sample of photo points for the remainder. Generally, this combination of areas will provide a more accurate estimate, although the variance computation will not show the improvement.

The variance of the grand totals can be calculated as the sum of the compartment variances. Forest totals could be estimated somewhat more precisely if counts of photo points and acres were totaled over the compartments first and these totals used in place of  $X_p$ ,  $T_p$ , and  $F_{hp}$  in the formulae for the compartment total and its variance. However, an important feature of this inventory design is its capability for including the results of additional stand examinations as they are accomplished. These additional data will be entered at the compartment record level. Hence, compilation procedures will be more easily handled if totals and variances are aggregated upward from that level.

In a similar way, statistics for various combinations of compartments can be compiled readily by adding totals and variances for the included compartments.

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